Efficient Multiple and Predicate Dispatching

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Abstract
The speed of message dispatching is an important issue in the overall performance of object-oriented programs. We have developed an algorithm for constructing efficient dispatch functions that combines novel algorithms for efficient single dispatching, multiple dispatching, and predicate dispatching. Our algorithm first reduces methods written in the general predicate dispatching model (which generalizes single dispatching, multiple dispatching, predicate classes and classifiers, and pattern-matching) into ones written using a simpler multimethod dispatching model. Our algorithm then computes a strategy for implementing multiple dispatching in terms of sequences of single dispatches, representing the strategy as a lookup DAG. Finally, our algorithm computes an implementation strategy separately for each of the single dispatches, producing for each dispatch a dispatch tree, which is a binary decision tree blending class identity tests, class range tests, and table lookups. Our algorithm exploits any available static information (from type declarations or class analysis) to prune unreachable paths from the lookup DAG, and uses any available dynamic profile information to minimize the expected time to search the dispatch trees. We measure the effectiveness of our dispatching algorithms on a collection of large Cecil programs, compiled by the Vortex optimizing compiler, showing improvements of up to 30% over already heavily optimized baseline versions.

1 Introduction
The speed of method dispatching is an important issue in the overall performance of object-oriented programs. Previous work on techniques for efficient dispatching has typically focused on singly dispatched languages; a few techniques have been proposed for multiply dispatched languages, and almost none for languages with other kinds of dispatching semantics such as predicate classes [Chambers 93b], classifiers [Hamer et al. 90, Mugridge et al. 91], or predicate dispatching [Ernst et al. 98] (which unifies and generalizes single dispatching, multiple dispatching, predicate classes, and pattern matching).

In this paper, we present an algorithm for constructing time- and space-efficient dispatch functions for the predicate dispatching model, including novel techniques for implementing single and multiple dispatching. Our algorithm consists of three main phases:

- The first stage reduces the general predicate dispatching model (including predicate classes, classifiers, and some features of pattern-matching) to the simpler task of multiple dispatching.
- The second stage determines the strategy for performing multiple dispatching, reducing multiple dispatching to sequences of single dispatches; the lookup DAG data structure represents this strategy. Compared to previous algorithms for multimethod dispatching and pattern-matching, our algorithm handles the more general predicate dispatching model, can select an appropriate order in which to test the classes of dispatched forms (not just left-to-right), can skip tests of forms through those paths where the test does not affect the outcome of dispatching (not just testing all forms along all paths), and can exploit available static information to skip tests whose outcomes are statically determined (supporting both statically and dynamically typed languages).
- The third stage determines the implementation of each of the single dispatches in the lookup DAG, customized to the properties of that particular dispatch. Each single dispatch is implemented using a dispatch tree, a binary decision tree blending class identity tests, class identifier inequality tests, and one-dimensional array lookups. Compared to previous algorithms for implementing single dispatching, our algorithm can effectively blend multiple different strategies to individually match the characteristics of each dispatch and balance speed against space consumption (not just applying a single strategy uniformly to all dispatches in the program) and can exploit available dynamic profile data.

We have implemented our algorithm in the context of the Vortex optimizing compiler [Dean et al. 96, Chambers et al. 96]. While our algorithm can be applied in several different ways, in our initial implementation we apply our algorithm when the whole program is available (roughly, at link time) to generate statically a single dispatch function for each generic function, shared by all sites calling that function. We assessed the effectiveness of our algorithm when used in this way on several large benchmark programs written in Cecil [Chambers 92, Chambers 93a] (a purely object-oriented language including multimethods and predicate classes, a large subset of the predicate dispatching model). We measured the speed of execution and the space consumed by dispatch functions, comparing different variations of our algorithm and the previous dispatch implementation based on dynamically generated call-site-specific polymorphic inline caches [Hölzle et al. 91]. We observed speed-ups of up to 30%, with greater speed-ups for larger programs. In our experiments, we applied Vortex's full range of optimizations, so that the benefits we observe for our dispatch algorithm are on top of the benefits already obtained by eliminating or inlining dispatches through other optimizations. Compared to previous work proposing dispatching algorithms, our
study uses large benchmark programs (not microbenchmarks or randomly generated program structures), measures not only the space consumed but also the speed of dispatching, and measures real execution times for a full program, including all caching effects (not just simulations or cache-miss-free microbenchmarks).

In the next section, we review the predicate dispatching model and explain how our algorithm reduces this model to an augmented multimethod dispatching model. In section 3, we describe our algorithm for constructing the lookup DAG representation of the strategy for multiple dispatching, and we compare our algorithm to related work on dispatching. In section 4, we describe our algorithm for determining the low-level implementation of each single-dispatch node of the lookup DAG, and we compare our algorithm to related work on single dispatching. In section 5, we present our results, and in section 6, we discuss the contributions and future work of our algorithm. We conclude with a discussion of contributions and future work in section 6.

2 Implementing Predicate Dispatching

The next subsection describes the predicate dispatching model, focusing on those aspects most relevant to the issues of efficient implementation; an earlier paper argues for its usefulness and generality [Ernst et al. 98]. Subsections 2.2 and 2.3 explain how to convert a program in predicate dispatching form into one using just an augmented multiple dispatching form.

2.1 Predicate Dispatching

In the predicate dispatching model, a generic function includes a collection of dynamically overloaded methods. (Each of the methods that are logically in a common generic function may physically be defined in a separate class, module, or file.) Each method is defined by a predicate expression giving the method's applicability and specificity and a method body specifying the code to run if the method is invoked. The grammar in Figure 1 defines the structure of a generic function, as viewed by the compiler after collecting together all the methods in that generic function in the program and renaming the formal parameters of all methods in the generic function to be the same. An example generic function, contrived to illustrate the main situations during compilation into a lookup DAG, is shown at the top of Figure 3.

When a generic function is applied to a tuple of argument objects, the predicate of each method in the generic function is evaluated, in an environment binding the generic function's formals to the argument objects, to determine if that method applies to the arguments. The different kinds of predicates are evaluated as follows:

- \texttt{Expr\textsubscript{Class}} is true iff evaluation of \texttt{Expr} in the current environment yields an object that is an instance of \texttt{Class} or some subclass of \texttt{Class}.
- \texttt{test\ Expr} is true iff evaluation of \texttt{Expr} in the current environment yields the true object.
- \texttt{Name := Expr} is always true, but updates the current environment to bind \texttt{Name} to the result of evaluating \texttt{Expr}.
- \texttt{not Pred} is true iff \texttt{Pred} is not. Any updates to the environment by \texttt{Pred} are ignored.
- \texttt{Pred1 or Pred2} is true iff \texttt{Pred1} is true (yielding an enhanced environment if \texttt{Pred1} contains any name bindings) and \texttt{Pred2} is true in the enhanced environment. \texttt{Pred2} is only evaluated if \texttt{Pred1} is true, allowing expressions in \texttt{Pred2} to be defined only for cases where \texttt{Pred1} succeeds. For example, \texttt{test\ x \neq 0 and\ test\ y/x \geq 5} exploits the required evaluation order, as does \texttt{list\ head\ int}.
- \texttt{Pred1 or Pred2} is true iff \texttt{Pred1} is true or \texttt{Pred2} is true. There are no constraints on the evaluation order of \texttt{Pred1} and \texttt{Pred2} (either or both of \texttt{Pred1} and \texttt{Pred2} may be evaluated, using short-circuit evaluation if desired), and any updates to the environment by \texttt{Pred1} or \texttt{Pred2} are invisible outside of \texttt{Pred1} or \texttt{Pred2}, respectively.\footnote{In this paper, we are ignoring the possibility of bindings escaping disjunctions [Ernst et al. 98].}
- \texttt{true} is always true.

After collecting the set of applicable methods, the unique most-specific method is identified and then invoked. One method \texttt{ml} is deemed at least as specific as another method \texttt{m2}, written \texttt{ml \sqsubseteq m2}, exactly when \texttt{m1}'s predicate implies \texttt{m2}'s. The different cases in which one predicate implies another, determined statically based on the structure of the two predicates, are as follows:

- \texttt{Expr\textsubscript{Class1}} implies \texttt{Expr\textsubscript{Class2}} iff \texttt{Expr1} is the same as \texttt{Expr2} and \texttt{Class1} is equal to or a subclass of \texttt{Class2}. Two expressions are the same if their abstract syntax
Example class hierarchy:

```
object A;
object B isa A;
object C;
object D isa A, C;
```

Example source generic function:

```
gf Fun(f1, f2)
when f1@A and t := f1.x and t@A and (not t@B) and f2.xeC and test(f1.y = f2.y) { ...m1... }
when f1.x@B and ((f1@B and f2.x@C) or (f1@C and f2@A)) { ...m2... }
when f1@C and f2@C { ...m3... }
when f1@C { ...m4... }
```

Assumed static class information for expressions (StaticClasses):

```
f1: AllClasses: {A,B,C,D}
f2: AllClasses = {A,B,C,D}
f1.x: AllClasses = {A,B,C,D}
f2.x: Subclasses(C) = {C,D}
f1.y = f2.y, bool = {true,false}
```

Canonicalized dispatch function:

```
df Fun(f1, f2)
  (f1@A and f1.x@A and f1.x@B and (f1.y = f2.y) @ true) => m1 {c1}
or (f1.x@B and f1@B) => m2 {c2}
or (f1.x@B and f1@C and f2@A) => m2 {c3}
or (f1@C and f2@C) => m3 {c4}
or (f1@C) => m4 {c5}
```

Canonicalized expressions and assumed evaluation costs:

```
e1 = f1 (cost=1)
e2 = f2 (cost=1)
e3 = f1.x (cost=2)
e4 = f1.y = f2.y (cost=3)
```

Constraints on expression evaluation order (ignoring transitively implied constraints):

```
e1 \rightarrow_{Exp} e3; e3 \rightarrow_{Exp} e1; e1, e3 \rightarrow_{Exp} e4
{e1, e3} \rightarrow_{Exp} e4
```

Figure 3: Predicate Dispatching Example

Trees are isomorphic, after renaming formals consistently and (conceptually) replacing locally bound names with their definitions, this conservative definition of equivalence retains decidability of predicate dispatching.

- test Expr1 implies test Expr2 iff Expr1 is the same as Expr2.
- Since it is always true, any predicate implies Name := Expr.
- not Pred1 implies not Pred2 iff Pred2 implies Pred1.
- Pred1 and Pred2 implies both Pred1 and Pred2.
- A predicate Pred implies both Pred or Pred and Pred1 or Pred2.
- Any predicate implies true.

(To be complete up to equivalence of expressions, predicates should be converted into disjunctive normal form before applying the rules for not, and, and, or.) It is a message-not-understood error if no applicable methods are found, and it is a message-ambiguous error if no applicable method is the unique most-specific one. Formal dynamic and static semantics for predicate dispatching, along with a sound and complete static typechecking algorithm for determining whether invocation of any given generic function can ever lead to a message-not-understood or message-ambiguous error, have been specified [Ernst et al. 98].

The predicate dispatching model includes many existing dispatch mechanisms as restricted cases. Every object-oriented language includes the notion of generic function, either explicitly (as in Common Lisp [Bobrow et al. 88, Steele Jr. 90]) and C++ [Stroustrup 91, Java [Gosling et al. 96], Eiffel [Meyer 92], and Modula-3 [Nelson 91]). In Cecil and Smalltalk, a generic function corresponds to all the methods having the same message name and number of arguments. In C++, Java, Eiffel, and Modula-3, a generic function roughly corresponds to all the methods having the same name, number of arguments, and static argument types; more precisely, a generic function is created whenever a class contains a method declaration that does not override any inherited method declaration, and all methods that override the introducing method declaration are included in that method's generic function. Each dynamically dispatched call site applies a particular generic function to its arguments, selecting the single most-specific method in the generic function to invoke.

Many previous dispatching models correspond to special idiomatic uses of predicates:

- With single dispatching, a method m in a class Cm would be modeled with a predicate of the form se1@Cm, where se1 is the name of the generic function's first formal.
- With multiple dispatching, a multimethod m with k arguments that specializes its i'th formal formal1 to the class Cmi would be modeled with a predicate of the form formal1@@Classmi and ... and formalk@Classmk. Omitting a formal1@Classmi conjunct leaves that argument position unspecialized for that method; different methods can specialize on different argument positions.
- With predicate subclasses [Chambers 93b], if a predicate class PredClass is declared to be a subclass of some superclass
Class, then whenever an instance of Class satisfies some additional boolean predicate Test, the instance is considered to also be a subclass of PredClass. Typically PredClass has a set of methods that override Class's, which take precedence for instances that dynamically satisfy the Test predicate. A method \( m \) whose formal \( \text{formal}_i \) specializes on the predicate class PredClass can be modeled by replacing \( \text{formal}_i \) with \( \text{formal}_i \) for PredClass and \( \text{test} \) Test. Classifiers ([Hamers et al. 90, Mugridge et al. 91] and modes [Taivalsaari 93]) can be modeled with similar techniques [Ernst et al. 98].

- With pattern-matching, as found in languages like ML [Milner et al. 97] and Haskell [Hudak et al. 92], a function case can be defined for certain arguments based on their value or datatime structure, possibly examining the value or structure of subcomponents. Conjunctions can be used to test the properties of multiple arguments, as with multimethods. Tests on the arbitrarily nested subcomponent \( a \text{. nested \_ component} \) of an argument \( \text{formal}_i \) can be modeled by using \( \text{formal}_i \text{. nested \_ component} \) as the expression being tested. Tests for an expression \( \text{Expr} \) having a particular value can be modeled either by adopting a prototype-based language model (in which case "values" and "classes" are tested uniformly using \( \text{Expr} @ \text{Value} \) clauses), or by using the conjunction

\[
\text{Expr} @ \text{Class} \text{Value and test(Expr = Value)}
\]

where \( \text{Class} \text{Value} \) is the class of \( \text{Value} \). Names := \( \text{Expr} \) bindings can be used to give names to subcomponents, for use in the body of the method. (Compared to pattern-matching like that in ML and Haskell, predicate dispatching confers the additional benefit of inheritance of cases from superclasses to subclasses, the ability to add new cases to handle new subclasses without modifying existing cases or declarations, automatic ordering of cases based on specificity of patterns as opposed to textual order of cases, and the ability to reference a bound variable in later patterns (non-linear patterns)).

- Boolean guards on patterns, as found in Haskell, correspond to additional test predicates, potentially over multiple formals and local bindings.

In addition to modeling many previous dispatching mechanisms, new kinds of dispatching can be specified under predicate dispatching, including general disjunctions and negations of tests as well as combinations of primitives not previously supported, such as subclass testing combined with testing of arbitrary subcomponents.

### 2.2 Canonicalization of Dispatching Predicates

The first step of our algorithm converts a generic function using the predicate dispatching model into a simpler, canonical form based on an augmented multiple dispatching model, specified in Figure 2. In essence, this grammar represents all the predicates of all the methods of a generic function in disjunctive normal form, i.e., a disjunction of conjunctions of possibly negated \( \text{Expr} \text{Class} \) atomic clauses, using a standard algorithm. (It is possible for this conversion to grow the size of the predicates exponentially, but we do not expect this in practice, nor can it happen for the restricted cases of single dispatching, multiple dispatching, and predicate classes, which lack disjunctions and negations.)

1. Replace all test \( \text{Expr} \) clauses with \( \text{Expr} @ \text{True} \) clauses, where True is the (possibly artificial) class of the true object.
2. Remove all Name := \( \text{Expr} \) clauses and replace references to Name in later \( \text{Expr} \) expressions with \( \text{Expr} \). (This replacement can be done by sharing the single \( \text{Expr} \) parse tree in all referencing \( \text{Expr} \) trees, so this rewriting does not increase the size of the predicate expressions.)
3. Convert each method's predicate into disjunctive normal form, i.e., a disjunction of conjunctions of possibly negated \( \text{Expr} \text{Class} \) atomic clauses, using a standard algorithm. Our algorithm converts a regular generic function \( GF \) into a canonical dispatch function \( HF \) in the following steps:

1. Replace all test \( \text{Expr} \) clauses with \( \text{Expr} @ \text{True} \) clauses, where True is the (possibly artificial) class of the true object.
2. Remove all Name := \( \text{Expr} \) clauses and replace references to Name in later \( \text{Expr} \) expressions with \( \text{Expr} \). (This replacement can be done by sharing the single \( \text{Expr} \) parse tree in all referencing \( \text{Expr} \) trees, so this rewriting does not increase the size of the predicate expressions.)
3. Convert each method's predicate into disjunctive normal form, i.e., a disjunction of conjunctions of possibly negated \( \text{Expr} \text{Class} \) atomic clauses, using a standard algorithm. (It is possible for this conversion to grow the size of the predicates exponentially, but we do not expect this in practice, nor can it happen for the restricted cases of single dispatching, multiple dispatching, and predicate classes, which lack disjunctions and negations.)
4. Replace not (\( \text{Expr} \text{Class} \)) clauses with \( \text{Expr} @ \text{Class} \) clauses.
5. Place each method's predicate into canonical form by replacing each conjunction \( \text{Conjunction} \Rightarrow \text{m} \).
6. Form the disjunction of all the individual methods' canonicalized predicates, flattening out any resulting disjunctions of disjunctions to recover disjunctive normal form.

Our algorithm exploits available static information about the possible classes of expressions to reduce the size of the canonicalized dispatching predicate. Available static information is represented by a function \( \text{StaticClasses} : E \rightarrow \text{AllClasses} \) that represents for each expression \( e \in E \) (where \( E \) is the set of expressions occurring in the predicates of the methods in the generic function) the set of possible classes (a subset of \( \text{AllClasses} \), the set of all classes in the program) of which the result of evaluating \( e \) may be a direct instance. \( \text{StaticClasses} \) can be derived from static type declarations (if in a statically typed language) and/or from automatic static class analysis. For a dynamically typed language with no static analysis, a trivial \( \text{StaticClasses} \) function that maps each expression to \( \text{AllClasses} \) can be used. The following canonicalization steps exploit static class information:

7. Remove all atomic tests that are guaranteed to be true by static class information. In particular, remove atoms of the form \( \text{Expr} @ \text{Class} \) where \( \text{StaticClasses}(\text{Expr}) \subseteq \text{Subclasses(Class)} \), the set of all subclasses of Class (including Class itself). Similarly, remove all atoms of the form \( \text{Expr} @ \text{Class} \) where \( \text{StaticClasses}(\text{Expr}) \cap \text{Subclasses(Class)} = \emptyset \).
8. Remove all conjunctions containing atomic tests that are guaranteed to be false by static class information. In particular, remove all conjunctions containing atoms of the form \( \text{Expr} @ \text{Class} \) where \( \text{StaticClasses}(\text{Expr}) \cap \text{Subclasses(Class)} = \emptyset \). Similarly, remove all conjunctions containing atoms of the form \( \text{Expr} @ \text{Class} \) where \( \text{StaticClasses}(\text{Expr}) \subseteq \text{Subclasses(Class)} \).

A final clean-up step merges any duplicate conjunctions:

9. Replace the set of all cases of the form \( \text{Conjunction}_1 \Rightarrow \text{Method}_1 \ldots \text{Method}_m \) having the same \( \text{Conjunction}_1 \) with the single case \( \text{Conjunction}_1 \Rightarrow \text{Method}_1 \ldots \text{Method}_m \).

Our algorithm converts a regular generic function \( GF \) into a canonical dispatch function \( HF \) in the following steps:

1. Replace all test \( \text{Expr} \) clauses with \( \text{Expr} @ \text{True} \) clauses, where True is the (possibly artificial) class of the true object.
2. Remove all Name := \( \text{Expr} \) clauses and replace references to Name in later \( \text{Expr} \) expressions with \( \text{Expr} \). (This replacement can be done by sharing the single \( \text{Expr} \) parse tree in all referencing \( \text{Expr} \) trees, so this rewriting does not increase the size of the predicate expressions.)
3. Convert each method's predicate into disjunctive normal form, i.e., a disjunction of conjunctions of possibly negated \( \text{Expr} \text{Class} \) atomic clauses, using a standard algorithm. (It is possible for this conversion to grow the size of the predicates exponentially, but we do not expect this in practice, nor can it happen for the restricted cases of single dispatching, multiple dispatching, and predicate classes, which lack disjunctions and negations.)

1. This model for multimethods applies to languages like Cecil and Dylan that treat all argument positions uniformly, since implication between conjunctions is insensitive to the order of conjuncts. Languages like Common Lisp that prioritize earlier arguments over later arguments are not supported directly by the predicate dispatching model.
A preorder is a reflexive, transitive, but not necessarily antisymmetric relation. If there are several methods associated with a conjunction, and no one of these methods overrides all others, then this conjunction will lead to a message-ambiguous error at runtime. Our lookup DAG construction algorithm will handle this situation, so we do not check for it when building the canonical dispatch tree.

A preorder is a reflexive, transitive, but not necessarily antisymmetric binary relation. A partial order is an antisymmetric preorder.

This canonicalization process in effect reduces the problem of the general predicate dispatching model to the simpler multimethod dispatching model; indeed, canonicalization is largely the identity function on the subset of predicate dispatching corresponding to multimethod dispatching (conjunctions of subclass tests of formals). Each Case disjunct in the canonical dispatch tree is analogous to a multimethod, and each Expr being tested by the case is analogous to one of the multimethod's formals.

### 2.3 Constraints on Expression Evaluation

Our algorithm also computes constraints on the order of evaluation of the Exprs in the canonical dispatch predicate. According to the predicate dispatching model as described in subsection 2.1, Exprs must have no externally visible side-effects, and disjuncts can be evaluated in any order or even have their evaluations interleaved, but programmers are allowed to depend on order of evaluation of conjuncts where successful outcomes of earlier conjuncts (e.g. \( x \neq 0 \)) ensure that later conjuncts do not encounter errors during evaluation (e.g. \( y/x > 5 \)). Our algorithm could be conservative and assume that all conjuncts must be evaluated left-to-right, but in many cases it can be shown that expressions need not be evaluated in this order, and our algorithm will be able to produce faster, smaller dispatch functions if it can reorder conjuncts where safe.

To represent a conservative approximation to the minimal constraints on evaluation order that the lookup DAG must respect, our algorithm computes a partial order \( \leq_{\text{Expr}} \) over \( \mathcal{E} \), the set of all Exprs in the canonical dispatch predicate; \( e_1 \leq_{\text{Expr}} e_2 \) if \( e_1 \) may need to be evaluated before \( e_2 \) to ensure that \( e_2 \) does not fail unexpectedly. This partial order is built in two steps. First, a preorder \( \Rightarrow_{\text{Expr}} \) over \( \mathcal{E} \) is defined as the transitive closure of direct orderings \( e_1 \Rightarrow_{\text{Expr}} e_2 \). A direct ordering \( e_1 \Rightarrow_{\text{Expr}} e_2 \) holds if \( e_1 \) and \( e_2 \) both appear in some conjunction, \( e_1 \) appears before \( e_2 \) in that conjunction, and the compiler cannot prove that evaluation of \( e_2 \) would not encounter an error even if the test of \( e_1 \) is false (for example, simple references to formals are known statically to be evaluable in any order).

A cycle is formed in our preorder if \( e_1 \) is evaluated before \( e_2 \) in one conjunction, but \( e_2 \) is evaluated before \( e_1 \) in some other conjunction. A cycle in the constraints on expression evaluation seems to imply that there is no safe order in which to evaluate expressions, but there always exist legal orders, such as left-to-right evaluation of expressions within conjunctions, and conjunctions themselves evaluated in any order. Hence, such cycles must be an overly conservative description of the required constraints, caused by blindly merging constraints across disjuncts. In fact, since it is legal to evaluate either \( e_1 \) or \( e_2 \) first (depending on which conjunction is evaluated first), none of the constraints in a cycle is required for safe expression evaluation (the transitive closure must be formed before dropping any cyclic constraints, however). The final partial order \( \leq_{\text{Expr}} \) is derived from the preorder \( \Rightarrow_{\text{Expr}} \) by ignoring any cyclic orderings: \( e_1 \leq_{\text{Expr}} e_2 \) if \( e_1 = e_2 \) or \( e_1 \Rightarrow_{\text{Expr}} e_2 \) but not \( e_2 \Rightarrow_{\text{Expr}} e_1 \).

For example, in the following predicate:

\[
(e_1 \text{ and } e_2 \text{ and } e_3 \text{ and } e_4) \text{ or } (e_5 \text{ and } e_3 \text{ and } e_2 \text{ and } e_6)
\]

the following \( \Rightarrow_{\text{Expr}} \) relation results:

\[
(e_1,e_5) \Rightarrow_{\text{Expr}} (e_2,e_3) \Rightarrow_{\text{Expr}} (e_4,e_6)
\]

After dropping the cyclic constraints between \( e_2 \) and \( e_3 \), the following final \( \leq_{\text{Expr}} \) relation results:

\[
(e_1,e_5) \leq_{\text{Expr}} (e_2,e_3) \leq_{\text{Expr}} (e_4,e_6)
\]

Figure 3 shows the initial preorder and final partial order over expression evaluation for the running example. The preorder is derived from the order of expression evaluation in the various cases of the canonicalized dispatch function, ignoring orderings between tests of independent formals. Because \( f_1 \)'s class is tested before \( x \cdot x \) is evaluated in one case, but afterwards in another case, they can legally be evaluated in either order, and the final partial order reflects ignoring the cyclic constraints in the initial preorder.

### 3 Implementing Multiple Dispatching

Once each generic function is in canonical form, our algorithm determines the high-level strategy for selecting the right method to invoke, encoding its decisions in a lookup DAG whose interior nodes represent individual single dispatches. Any available information about the possible classes of the arguments to the generic function, such as determined by static type or class declarations [Johnson 86] or static class analysis [Johnson et al. 88, Chambers & Ungar 90, Plevyak & Chien 94, Fernandez 95, Agesen 95, Toman et al. 95, Gro佛 et al. 97, DeFunw et al. 98], is exploited to produce a faster, smaller lookup DAG.

The next subsection defines the lookup DAG structure. Subsections 3.2 through 3.4 present the steps in our algorithm to construct the lookup DAG. Subsection 3.5 compares our algorithm with other algorithms for multimethod dispatching and compilation of pattem-matching.

#### 3.1 The Lookup DAG

A lookup DAG \( G=(\mathcal{N}, \mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}, n_0 \in \mathcal{N}) \) is a rooted, directed acyclic graph, representing a decision “tree” but with identical subtrees shared to save space. Each interior node \( n \in \mathcal{N} \) in the lookup DAG has a set of outgoing edges \( n.\text{edges}=[(n,r)\in\mathcal{E}] \) and is labeled with an expression \( n.\text{expr} \in \text{Expr} \), while each leaf node \( n \in \mathcal{N} \) in the lookup DAG is labeled with a method \( n.\text{method} \), which is either a user-specified method or one of the two error methods \( \text{mnot-underspecified} \) and \( \text{mambiguous} \). Each edge \( e=(n,r) \in \mathcal{E} \) in the lookup DAG has a source node \( e.\text{source} \), a target node \( e.\text{target} \), and is labeled with a class \( e.\text{class} \). Figure 4 shows the lookup DAG our algorithm constructs for the example from Figure 3, where a circle represents an interior node \( n \) (labeled with \( n.\text{expr} \)), a box represents a leaf node \( n \) (labeled with \( n.\text{method} \)), an arrow represents a set of edges \( (e_1, \ldots, e_k) \) all of which have the same source and target nodes (labeled with the various \( e_i.\text{class} \) values of the edges \( e_i \)), and the entry arrow marks the root node \( n_0 \). (The sets subscribing nodes and the dashed circle are used during construction of the lookup DAG, as described later.)
To perform dispatching using a lookup DAG, evaluation follows a path through the DAG, beginning with the root node $n_0$ in an environment where the names of the generic function’s formals are bound to the corresponding actuals for the call being dispatched. To evaluate an interior node $n$, its expression $n.expr$ is evaluated to produce a result $v$, the node’s outgoing edges $n.edges$ are searched for the unique edge $e$ whose label $e.class$ is the class of $v$ (evaluation fails if no such edge is found), and the selected edge’s target node $e.target$ is evaluated recursively. To evaluate a leaf node $n$, its method $n.method$ is returned.

## 3.2 Construction of the Lookup DAG

Our algorithm for constructing the lookup DAG for a canonicalized dispatching function $DF$ is given in Figure 5. The heart of the algorithm is the function buildSubDag which builds a node and all its subnodes, given a set of candidate cases ($cs$) and a set of expressions in the candidate cases remaining to be evaluated ($es$). The root node of the DAG corresponds to the set of all cases in $DF$ and the set of all expressions in these cases. The table Memo memoizes calls to buildSubDag so that at most one node for a given set of cases and remaining expressions will ever be constructed, with that node and its successors shared by all earlier nodes leading to it.

When building a new node for a particular set of cases and remaining expressions, if the set of remaining expressions is empty, then the outcome of method lookup has been determined. A leaf node is constructed, and its target (or error) method computed (using computeTarget) from the most specific method(s) (under the method overriding partial order $\preceq_{\text{Method}}$) in the set of methods associated with the cases that reach the leaf. If, on the other hand, the set of remaining expressions is not empty, then more dispatching is necessary. An interior node is constructed, and one of the remaining expressions selected (via pickExpr) as the next to evaluate at this interior node. For each of the possible classes resulting from this evaluation, the subset of the input cases is computed whose atomic tests on the selected expression all succeed for that class (by targetCases). The set of expressions still to be evaluated for that subset is computed by restricting the set of unevaluated remaining expressions to those mentioned in the computed subset of cases. Finally, a sub-DAG for the reduced set of cases and remaining expressions is constructed recursively, and finally an edge is created to connect the interior node to the target sub-DAG.

The pickExpr function is responsible for selecting the next expression to evaluate from the set of remaining expressions. The one correctness constraint on pickExpr’s behavior is that it can only pick expressions that are not required to follow (according to the $\preceq_{\text{Expr}}$ partial order described in section 2.3) any other expressions still remaining to be evaluated; these allowed choices are collected into legalEs. Within this constraint, we wish to select an expression that will minimize the expected time to proceed through the rest of the lookup DAG. As a heuristic approximation,
function buildLookupDag(DF: canonical dispatch function) : lookup DAG =
create empty lookup DAG G
create empty table Memo
cs:= set of Case := Cases(DF)
G.root := buildSubDag(cs, Exprs(cs))
return G

function buildSubDag(cs: set of Case, es: set of Expr) : set of Case =
n := node
if (cs,es) -> n ∈ Memo then return n
if cs = Ø then
  n := create leaf node in G
  n.method := computeTarget(cs)
else
  n := create interior node in G
  n.expr := pickExpr(es, cs)
  n.method := computeTarget(cs)
  foreach class ∈ StaticClasses(expr) do
    cs' := targetCases(cs, expr, class)
    es' := es - Expr(cs')
    n' := buildSubDag(cs', es')
    e := create edge from n to n' in G
    e.class := class
  end for
end if
add (cs,es) -> n to Memo
return n

function computeTarget(cs: set of Case) : Method =
methods := {Method | Method ∈ Methods(cs)}
if |methods| = 0 then return not understood
if |methods| > 1 then return ambiguous
return single element of methods

function pickExpr(es: set of Expr, cs: set of Case) : Expr =
legal := {expr | expr ∈ es}
discriminating := {expr | expr ∈ legal and avgNumTargetCases(cs, expr) is minimal}
cheap := {expr | expr ∈ discriminating and expr has minimal evaluation cost}
return any member of cheap

function avgNumTargetCases(cs: set of Case, expr: Expr) : number =
|StaticClasses(expr)|

function targetCases(cs: set of Case, expr: Expr, class: Class) : set of Case =
{ c ∈ cs | class ∈ classesPassingTest(c, expr) }

function classesPassingTest(c: Case, expr: Expr) : set of Class =
if expr ∈ Expr(c) then return { atom | atom ∈ Class(c), Expr(atom) = expr }
else return AllClasses

function Cases(df Name(Name_1, ..., Name_n) Case_1 or ... or Case_p) = {Case_1, ..., Case_p}
function Methods(Conjunction => Method_1 ... Method_n) = {Method_1, ..., Method_n}
function Atoms(Conjunction => Method_1 ... Method_n) = Atoms(Conjunction)
function Expr(Expr@Class) = Expr
function Expr(Expr@1Class) = Expr
function Subclasses(Class) = set of all subclasses of Class, including Class itself
value AllClasses = set of all classes in program

function Exprs(cs: set of Case) : set of Expr = Exprs(cs)
function Exprs(c: Case) : set of Expr = { Expr(atom) | atom ∈ Atoms(c) }

Figure 5: Lookup DAG Construction Algorithm
we pick an expression whose average number of remaining cases in successor nodes is minimized (discriminating es); nodes with no remaining cases are leaves, and nodes with fewer remaining cases are more likely to have shorter paths to leaves than nodes with more remaining cases. If ties still remain, we select an expression that is cheapest to evaluate, using some precomputed static estimate of the cost of evaluation (cheap es). pickExpr is the one place in our high-level algorithm that uses heuristics, and alternative heuristics could be substituted for these without affecting the correctness of our algorithm.

Figure 4 shows the lookup DAG constructed by our algorithm for the example from Figure 3; the two sets below each node correspond to the (cs,es) pair mapping to that node in the Memo table. At the start of the example, expression \( e_1 \) is picked as the first to evaluate, as it is the most discriminating of the legal expressions \( \{e_1,e_2,e_3\} \). \( e_4 \) is not legal as it must follow evaluation of other unevaluated expressions. Each of the possible classes for \( e_1 \) is considered, computing the subset of cases that apply to that outcome (cs') and the set of expressions remaining to be evaluated to distinguish those applicable cases (es'). For this example, all three possible classes lead to distinct nodes, which are then processed recursively and independently. Two of the nodes choose to evaluate expression \( e_2 \) next, while the third chooses to evaluate \( e_3 \). During subsequent recursive construction, the node labeled \( \{(e_4)\} \) is analyzed twice, and the sub-DAG starting with that node is shared. When reaching a leaf node, the set of applicable cases for that node is used to determine the set of applicable methods for that node, which in turn determines the target or error method to invoke for that leaf. If no applicable methods remain (i.e., if no cases reach that leaf node), the message-not-understood error method is used. If several applicable methods remain but one is more specific than all others (as in the nodes labeled \( \{c_{P,c}\} \) and \( \{c_{E,c}\} \)), then the most-specific method is selected as the target method. If multiple applicable methods remain but none is most-specific (as in the node labeled \( \{c_{P,E},c_{E,P}\} \) where \( c_{P,E} \) and \( c_{E,P} \) are applicable but neither is uniquely most-specific), then the message-ambiguous error method is used.

An important feature of our algorithm is that the order of evaluation of expressions can be selected according to heuristics, in pickExpr. Some previous work only considered left-to-right evaluation of multimethod arguments [Dussud 89, Chen et al. 94], but alternative orders of evaluation that test more-discriminating expressions first can produce smaller, faster dispatchers. Another important feature of our algorithm is that each of a node's successor subtrees is computed independently, with different subsets of expressions evaluated along different paths, possibly in different orders, as opposed to previous work which examined the dynamic classes of all formal in all cases. By recomputing the set of remaining expressions independently along each path, we can produce faster dispatch functions, as occurred in the example in Figure 4. We had previously observed the need for this ability in practice, where all methods in a generic function dispatch on the first argument, but only a few dispatch on additional arguments. By testing the additional arguments only on paths where those methods apply, most paths need only a single test to resolve dispatching to the target method. Previously developed techniques, including all array-based techniques, require testing every argument dispatched by any method on all invocations of the generic function.

The complexity of building the lookup DAG, assuming that calls to classesPassingTest are memoized, is \( O(D \cdot (C \cdot P + C \cdot E + P \cdot E + M)) \), where \( D \) is the number of nodes in the constructed lookup DAG, \( C \) is the number of concrete classes in the program, \( P \) is the size of the predicates (\( O(P) \), \( E \) is the number of expressions in the predicates (\( O(E) \)), and \( M \) is the number of target methods in the generic function. The number of nodes in the DAG, \( D \), is no worse than exponential in the number of classes in the program \( C \) and the number of expressions in the predicate \( E \), and this situation can occur if \( M \) or \( E \) is also exponential in \( C \), but we do not know if a tighter bound exists on the size of the lookup DAG if we assume that \( C, P, E, \) and \( M \) are all \( O(N) \), where \( N \) is the size of the program.\(^1\)

3.3 Postprocessing

Memoization of calls to buildSubDAG ensures that no two nodes in the lookup DAG have the same set of cases and remaining expressions. However, it is possible that two leaf nodes with different sets of cases still have the same target (or error) method, and those nodes can be safely merged without affecting correctness. After merging all leaf nodes having the same target method, if a merged leaf node's predecessor now has only a single successor node, it can be eliminated. Similarly, if merging leaf nodes into one causes two or more predecessor nodes to have the same successors (and edge labels leading to those successors), the predecessors can be merged. These removing and merging transformations can ripple backwards through the lookup DAG. It is difficult to perform these clean-ups as the lookup DAG is being constructed because it is hard to predict for two different sets of cases and remaining expressions whether the sub-DAGs constructed from them will be isomorphic and lead to the same target methods. Our algorithm instead performs these kinds of clean-up transformations in a single backwards pass over the lookup DAG after construction.

In the lookup DAG shown in Figure 4, there are two leaf nodes (circled with a dotted line) that are labeled with different cases but that both invoke method \( f_2 \). Postprocessing would merge these two nodes into a single node. For this example, no additional nodes can be merged.

3.4 Factoring Common Subexpressions

The final step in constructing an efficient lookup DAG is to factor common subexpressions in the expressions being evaluated in the lookup DAG. Common subexpressions across expressions may have been created as a result of eliminating Name := Expr clauses during the canonicalization of predicates described in section 2.2. They may also have appeared naturally in the original predicate expressions; since predicate expressions are written separately for separate methods, there may not have been any obvious source-level common subexpressions within individual method declarations, but rather subexpressions may have been common only across method declarations.

Our algorithm applies a variant of partial redundancy elimination (a well-studied traditional compiler technique [Morel & Renvoise 79]) to identify common subexpressions and to place computations of those subexpressions in the lookup DAG (possibly creating new intermediate nodes in the graph whose sole purpose is to evaluate some subexpression). Our variant of partial redundancy elimination can ensure that each subexpression is evaluated at most once along any path through the DAG. In effect, this transformation reinserts a minimal number of Temp := SubExpr clauses into the lookup DAG for any common subexpressions.

3.5 Comparison with Previous Work

Only a few approaches have previously been devised for efficient multimethod dispatching. Kiczales and Rodriguez [Kiczales & Rodriguez 90] describe a strategy where each generic function has

\(^1\) \( E \) could even be treated as bounded by a small constant, as is commonly assumed about the number of arguments to a function.
a corresponding hash table, mapping from the tuple of the dynamic classes of the specialized message arguments to the target multimethod. Dussud [Dussud 89] describes a strategy for TICLOS using a tree of hash tables per generic function, each hash table mapping the dynamic class of a particular argument to a nested subtree of hash tables that collectively test the remaining arguments, with leaf tables mapping to the target method. Dussud’s tree of single-dispatching hash tables can be viewed as a restricted, less optimized version of our lookup DAG, similar in that it factors out the results of tests of earlier arguments, but different in restricting each lookup DAG node to be implemented with a hash table (our algorithm can customize the implementation of each node separately, as discussed in section 4), producing distinct subtables for each class index in a table (unlike the sharing of successors by multiple class indices of a node and even across nodes), and requiring all dispatched arguments to be evaluated along all paths in a fixed left-to-right order. Kiczales and Rodriguez’s algorithm computes a single hash over a tuple of classes rather than a series of hashes over single classes as does our and Dussud’s algorithms; a single hash over a tuple may be faster if all arguments need to be tested, but if different methods test different numbers of arguments, then our more selective algorithm may perform better. Both hash-table-based schemes are intended to work on-line, filling the hash tables as the program runs and only recording entries for combinations of classes that occur in the actual program run. The first time a particular combination of argument classes is seen, a more expensive lookup algorithm is performed to fill in the table(s). In contrast, our algorithm computes a single dispatch function for all possible combinations of arguments in one step, in some cases producing a bigger and slower dispatch function than the hash-table-based approaches. We expect our algorithm to be applied on-line at static compile or link time, incurring no runtime cost as a result. The space cost of both of these dynamically filled hash-table-based schemes is proportional to the number of combinations of argument classes used during program execution, which in the worst case is \( N^k \), where \( N \) is the number of classes in the program and \( k \) is the number of specialized argument positions; the sharing of subtrees in our approach is likely to produce much more compact dispatchers in the common case. Only microbenchmark performance results were reported for these implementations.

Chen and colleagues [Chen et al. 94] developed an approach based on a decision DAG (which they describe as a finite-state automaton) per generic function. Each node in the decision DAG represents a test of a particular argument’s dynamic class, with each outgoing edge representing a different outcome; multiple dynamic classes can be represented by a single edge, if they invoke the same set of target methods under the same conditions. By sharing nodes, the space cost of this approach can be much lower than the hash-table-based approaches. Our algorithm was inspired by this earlier work, sharing its underlying DAG-based approach. Our algorithm generalizes Chen’s algorithm to support the predicate dispatching model, to test the class of forms in any order (not just left-to-right\(^1\)), to allow testing the classes of different subsets of forms on different paths through the DAG, and to allow the implementation of each node in the DAG to be customized independently (as discussed in section 4). Additionally, Chen’s algorithm was not empirically assessed for either time or space on any benchmarks, while our algorithm is assessed on a collection of large Cecil programs (as described in section 5).

In general, multimethod dispatching for a generic function of \( k \) dispatched arguments can be viewed as indexing into a \( k \)-dimensional matrix whose elements are the target (or error) methods to invoke for a lookup, assuming that the class of an object is represented by an integer in the range \([0..N]\). Since this matrix consumes \( N^k \) words of space, this approach to dispatching is not directly practical. To reduce the space costs, Amiel et al. [Amiel et al. 94] developed techniques for compressing the matrix by finding and merging identical submatrices of the matrix, at the cost of introducing an additional \( N \)-long helper array per dispatched argument. The end result is a system that performs \( k \)-one-dimensional array index operations and one \( k \)-dimensional matrix index operation for each dispatch, and consuming \( k \cdot N \cdot O(N^k) \) space for each generic function. The size of the asymptotically exponential term in the space complexity is critically dependent on the effectiveness of compression. In many situations compression can be good, but for binary methods like `eqv`, where most classes \( c \) define a \( c \cdot c \) multimethod case, the diagonal of the matrix is filled with mostly distinct entries, causing most planes to be unique and preventing effective compression. Amiel’s algorithm originally was only assessed in terms of space cost, and then only for randomly generated generic functions. Randomly generated generic functions may not resemble real-world generic functions; for example, the all-diagonal matrix is unlikely to be generated randomly. Later work [Dujardin et al. 98] used data taken from the Cecil system, but again only space costs were studied.

Pang et al. [Pang et al. 99] developed another \( k \)-dimensional matrix-based approach which can be viewed as an extension to Amiel’s compression technique. To save additional space, the various one-dimensional arrays are overlapped where possible using a row-displacement-style algorithm [Driesen 93, Driesen & Holzke 95]. They implemented their algorithm and compared its space consumption to several other matrix-based algorithms and a version of Chen’s algorithm, using data from the Cecil system. They also measured the dispatching speed of the various algorithms, but they used only microbenchmarks that essentially ensured that all memory references hit in the cache, giving a best-case measurement to the speed of the table-based techniques.

Dujardin [Dujardin 96] developed an approach using decision DAGs similar to Chen’s and our algorithms. Unlike Chen’s algorithm but like ours, Dujardin’s algorithm targeted unordered multimethods as in Cecil. Dylan, and the multimethod subset of predicate dispatching. Dujardin discussed selecting a testing order other than left-to-right, but did not present any specific heuristics. Dujardin’s algorithm implements each node’s dispatch with two array lookups, using one \( N \)-long helper array per dispatched argument and one compressed array per dispatch node, using the same kinds of table compression ideas as in Amiel’s algorithm. Dujardin used data from the Cecil system to compare the space cost of his decision DAGs against Amiel’s multidimensional matrix-based scheme. His algorithm produces trees with the same space cost as matrices for generic functions specialized on fewer than three arguments, but for generic functions with three or four specialized arguments (the maximum in his data set), dispatch DAGs are roughly half the size of the corresponding compressed matrices. There was no study of the impact on speed of the different dispatching mechanisms.

Pattern-matching is routinely compiled using techniques similar to our work [Peyton Jones 87], albeit for a subset of our generic function model lacking inheritance and arbitrary boolean tests. The main approach is to produce a DAG of constructor-tag tests, striving to examine each part of the data structure at most once, testing

\(^1\) The Chen decision DAG was aimed at Common Lisp-like languages with left-to-right argument prioritization, so a fixed left-to-right evaluation order made sense for the underlying dispatching model. Our predicate dispatching model and lookup DAG algorithm does not directly handle Common Lisp-like dispatching rules.
potentially different subsets of tags needed along different paths through the DAG, and sharing common sub-DAGs where possible. The compilation algorithms are typically described as source-to-source transformations, which get difficult to decipher when attempting to avoid redundant tests and to introduce shared sub-DAGs.

4 Implementing Single Dispatching

The lookup DAG constructed by the previous phase of our algorithm leaves unspecified how each interior node is to be implemented. Conceptually, each interior node performs a multi-way switch over the N possible classes of the expression being tested by the node. Several different techniques have been used previously to implement this multi-way switch:

- In many systems, including most statically typed singly dispatched languages, multi-way switches are implemented as lookups in an N-long array, assuming that the class of an object is encoded as an integer in the range \([0..N-1]\). This strategy is most efficient if \(N\) is large, most entries of the array are different, and the dynamic frequency distribution over the possible classes is flat (evenly distributed). Each of the \(k\) individual dispatches in \(k\)-dimensional matrix-based multimethod dispatching algorithms [Amiel et al. 94, Pang et al. 99] is formed from a pair of one-dimensional array lookups, further exacerbating the memory load bottleneck. As memory speeds increasingly lag CPU speeds, the memory load instruction(s) for the array lookup(s) can become a serious performance bottleneck.

- In some systems, typically early dynamically typed systems, multi-way switches are implemented as dynamically filled hash table lookups [Kiczales & Rodriguez 90, Dussud 89, Kachler & Krasner 83]. This strategy is efficient if it is hard to predict statically what classes will be used, and the dynamic frequency distribution over the possible classes is flat.

- In some systems, including several more recent dynamically typed systems, multi-way switches are implemented as linear searches through the possible classes, as in polymorphic inline caches [Hölzle et al. 91]. To make the linear search effective, dynamic profile information is used to order the tests in decreasing order of likelihood, typically by constructing the linear search code dynamically as the program runs. This strategy is efficient if the frequency distribution is highly skewed toward a few classes, as are many call sites in practice. Indeed, linear searching can outperform table-based lookups for highly skewed frequency distributions, since the code for comparing a register against a series of small integer constants is quick while the memory loads and indirect jumps of the table-based schemes can incur expensive pipeline stalls [Driesen et al. 95].

- In some systems, multi-way switches are implemented as a linear search through subclass tests [Chen et al. 94]. Instead of testing for individual classes, the set of all subclasses of a given class are tested as a unit, where all those subclasses branch to the same target node. Overlapping subclass tests must be performed bottom-up. This approach performs fewer tests in the worst case than linear search through individual classes, but the cost of a single subclass test is more expensive than a single class identity test (e.g., requiring at least an extra load instruction under the various strategies described by Vitek et al. [Vitek et al. 97]) and it is difficult to test for commonly occurring classes early if they have (perhaps infrequently occurring) subclasses that branch to different target nodes.

- In a few systems, multi-way switches are implemented using balanced binary search through the integer encodings of the possible classes [Nakamura et al. 96, Zendra et al. 97]. By taking only logarithmic time rather than linear time, worst-case dispatching is sped up, but previous systems have not exploited profile data to make expected time be better than logarithmic. Space costs can also be reduced over linear search or table lookups, if the number of target nodes is much smaller than the number of possible classes.

None of these techniques dominates all others under all circumstances, and most techniques are the best choice for some commonly occurring circumstance. Despite this mixed result, previous systems have picked a single dispatching mechanism and applied it universally to all dispatches in the system. Our algorithm instead crafts a multi-way dispatching code sequence for each node in the lookup DAG individually, producing what we call a dispatch tree. We assume that an object represents its class using an integer value, unique to that class, which we call the class ID. The dispatch tree constructed for an interior node first evaluates the node's expression, then loads the result object's class ID, and then performs a multi-way branch through a combination of equality tests (as in linear searches), less-than tests (as in binary searches), and one-dimensional array lookups, all based on the class ID. Our algorithm attempts to minimize the expected time to perform the multi-way branch, based on the expected cost in cycles of the different instructions making up the dispatch and the expected frequency distribution of the possible classes, derived either from dynamic profiles [Grove et al. 95] or simple static estimates. For example, given the high cost of memory references relative to integer computations, our algorithm will naturally choose modest-sized binary search trees involving only compare and branch instructions over even a single array lookup. Our algorithm also attempts to balance space against speed, avoiding choices that incur great space cost with only minimal speed benefit. By customizing the dispatch implementation to the individual characteristics of each node, we attempt to gain the advantages of most of the above approaches, often in combination, without suffering their disadvantages.

Hyafil and Rivest have shown that constructing an optimal decision tree from arbitrary tests is NP-complete [Hyafil & Rivest 76]. Hu and Tucker devised an \(O(N \log \lambda)\) algorithm for constructing optimal decision trees using only less-than tests [Hu & Tucker 71], and optimal decision trees using only equality tests or array lookups are easy to construct in linear time. Our problem, mixing three kinds of restricted tests, lies in between these extremes; we do not know whether our problem admits a polynomial-time optimal algorithm.

4.1 Class IDs and Frequencies

We assume that each class has a single associated integer class ID, unique over all classes, stored in each object that is an instance of the class. Our algorithm works correctly for any such assignment, but it will work better if the class IDs of all subclasses of a class are contiguous. If they are, then a pair of comparison operations can be used to implement a subclass test cheaply without additional memory operations. For systems with single inheritance, it is possible to arrange for this contiguity, but for multiple inheritance it is not always possible. In our implementation in Vortex, we simply assign class IDs to classes in a preorder, top-down, topological traversal of the inheritance graph; for tree-structured subgraphs of the inheritance graph, class IDs are assigned

---

1 Most singly dispatched languages implement tables stored with each class indexed by an integer identifier of the generic function being invoked. In this paper we consider only the transposed organization: tables stored with the generic function indexed by integer identifiers of the class.
Class IDs, Targets, and Frequencies:

<table>
<thead>
<tr>
<th>Class ID</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</tr>
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<td>6</td>
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<th>46</th>
<th>47</th>
<th>48</th>
<th>49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq.</td>
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<td>2</td>
<td>2</td>
<td>100</td>
<td>1</td>
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<td></td>
</tr>
</tbody>
</table>

Dispatch Tree and Intervals:

![Dispatch Tree Diagram]

Figure 6: Example for Dispatch Tree Construction

Our dispatch tree construction algorithm conceptually partitions this mapping into a set of intervals. An interval is a maximal subrange of integer class identifiers \([lo..hi]\) such that all IDs in the interval map to the same target state or are undefined (i.e., are not in the set of possible classes for this interior node). The intervals for the example class IDs and targets are shown at the bottom of Figure 6. Our algorithm first constructs a binary tree over these intervals, using equality and less-than tests on class IDs (as described in subsection 4.2.1) and then replaces selected subtrees with array lookups when profitable (as described in subsection 4.2.2).

4.2 Dispatch Tree Construction

The interior node's multi-way branch maps each of the \(N\) possible class IDs (the labels on the node's outgoing edges) to a target node. Our dispatch tree construction algorithm conceptually partitions this mapping into a set of intervals. An interval is a maximal subrange of integer class identifiers \([lo..hi]\) such that all IDs in the interval map to the same target state or are undefined (i.e., are not in the set of possible classes for this interior node). The intervals for the example class IDs and targets are shown at the bottom of Figure 6. Our algorithm first constructs a binary tree over these intervals, using equality and less-than tests on class IDs (as described in subsection 4.2.1) and then replaces selected subtrees with array lookups when profitable (as described in subsection 4.2.2).

4.2.1 Binary Tree Construction

The first step of our algorithm, shown in Figure 7, constructs a binary tree over intervals, guided by the expected frequency distribution, in an effort to minimize expected execution time for searching the binary tree. The resulting binary tree is either a leaf node containing a target lookup DAG node, or an internal node containing a test and two subtrees. The test in an internal node is either an equality or less-than test against a fixed class ID.
function buildDispatchTree (n: node): dispatch tree =
  map: integer -> node := \{ id + target \mid e ∈ n.edges ∧ Class ID(e.class) = id ∧ e.target = target \}
  sorted_ids: list of integer := list of all id ∈ dom(map), sorted in decreasing order of frequency(id)

  frequency: number := \sum_{id ∈ dom(map)} frequency(id)
  return buildDispatchSubTree(map, sorted_ids, frequency)

function buildDispatchSubTree (map: integer+node, sorted_ids: list of integer, frequency: number): dispatch tree =
  if |range(map)| = 1 then
    return new leaf(target) where target is single element of range(map)
  best_id: integer := first id ∈ sorted_ids
  if frequency(best_id) > frequency * 0.4 then
    best_target: node := map(best_id)
    true_subtree: dispatch tree := new leaf(best_target)
    false_subtree: dispatch tree := buildDispatchSubTree(map - {best_id}, best_target)
  else
    divide_id: integer, frequency_below: number, frequency_above: number := pickDivider(map)
    true_subtree: dispatch tree := buildDispatchSubTree(map whose domain is restricted to those < divide_id, sorted_ids restricted to those < divide_id, frequency_below)
    false_subtree: dispatch tree := buildDispatchSubTree(map whose domain is restricted to those ≥ divide_id, sorted_ids restricted to those ≥ divide_id, frequency_above)
  return new node(new test(<, divide_id), true_subtree, false_subtree)

function pickDivider (map: integer+node): (integer, number, number) =
  return id, frequency_below, frequency_above such that
  id: integer ∈ dom(map),
  id' : integer ∈ dom(map) is maximum such that id' < id,
  map(id) = map(id'),
  frequency_below: number := \sum_{id ∈ dom(map), id < divide_id} frequency(id)
  frequency_above: number := \sum_{id ∈ dom(map), id ≥ divide_id} frequency(id)
  |frequency_below - frequency_above| is minimal

Figure 7: Dispatch Tree Construction Algorithm

The heart of the algorithm, buildDispatchSubTree, operates recursively, given a mapping from class IDs to target lookup DAG nodes (map), a list of the class IDs sorted in decreasing order of frequency (sorted_ids), and the total frequency of the class IDs in the mapping (frequency). The algorithm checks for three possible cases at each recursive step:

1. If all class IDs map to the same target node, i.e., the map is a single interval, then a leaf node is returned.
2. If the most frequent class ID has a relative frequency above some threshold (Threshold, set to 40% in our current implementation), then an equality test against this ID is constructed, with a subtree constructed recursively for the remaining IDs if the equality test fails.
3. Otherwise, the class ID starting an interval is selected (by pickDivider) that most closely divides execution frequency in half. A less-than test against this ID is constructed, with the left and right subtrees constructed recursively from the class IDs less-than and greater-than-or-equal-to this ID, respectively.

To make the pickDivider calculations efficient, our implementation maintains an array of intervals for the class IDs in the domain of map, with each interval recording the prefix sum of the execution frequencies of all IDs less than or equal to its high ID.¹

This algorithm attempts to blend the strengths of linear searching and binary searching. Linear searching works well if the execution profile is strongly peaked, while binary searching works well when the profile is flat. When doing binary searching, by dividing the execution frequencies nearly in half, we hope to balance the weighted expected execution time of searching the two subtrees and thereby minimize the total expected search time for the whole tree.

The time to construct the binary decision tree using our algorithm is O(C²), where C is the number of possible classes of the expression being tested.

Figure 6 shows the results of this phase of the algorithm on the running example. In the first call to buildDispatchSubTree, class ID 14 has more than 40% of the total frequency, and so an equality test is created for it, and the remaining IDs processed recursively. In the remaining IDs, no single ID has more than 40% of the total frequency, and so the interval boundary that most evenly divides the remaining execution frequency is identified (21), a less-than test created at this ID, and the two subranges processed recursively. Within the right subrange, ID 31 now has more than 40% of the frequency within that subrange, and so another equality test is inserted. Binary tree construction proceeds in this manner.

¹ As an additional case, our implementation also checks for the case where all but one class ID maps to a given target, and the oddball class ID breaks up the other IDs into two disconnected pieces. In this situation, an equality test for the oddball class ID is constructed, selecting between two leaf nodes. If this special case were not included, and the oddball class ID wasn’t very frequent, then two less-than tests would be needed to separate the three intervals.
that correspond to subtrees, and many subranges that could make
previous phase, selectively replacing subtrees by arrays, has the
estimates) and taking time linear in the size of the original binary
tree. However, it only considers those few subranges of class IDS
t in our SPARC-based system, a compare-and-branch sequence is assumed
to improve the implementation of the dispatch
node by allocating an array with index domain equal to [lo..hi]
where lo and hi are the smallest and greatest class IDS handled by
that subtree, filling in the defined array elements with the
corresponding target lookup DAG node. (In our implementation, if
the target lookup DAG node n is itself a leaf DAG node holding a
target method n.method, we store the address of n.method in
the array element.)

Our algorithm works bottom-up over the binary tree, computing for
each interior node in the tree the expected execution time and actual
space costs for that subtree when represented as a binary tree and
when represented as an array. The time and space costs are
computed based on constants specifying the time and space costs
for the various machine instructions and data space used in the
implementation of the subtree, with the frequency distribution
being used to weight different paths through the decision tree. For
each node, we make a decision between implementing the decision
as a binary tree or as an array, comparing the expected speedup
against the expected space increase for switching to an array. Our
current implementation uses the following comparison, switching
to an array-based implementation if the comparison is true:

\[
\frac{t_{\text{tree}} - 1}{t_{\text{array}}} \cdot \frac{\text{frequency}}{s_{\text{array}}} \leq \frac{1}{s_{\text{tree}}} \cdot \text{space-speed-trade-off}
\]

In this equation, \(t_{\text{tree}}\) and \(t_{\text{array}}\) are the expected times to search the
tree and into the array, respectively, and \(s_{\text{tree}}\) and \(s_{\text{array}}\) are the
space in space to represent the tree and array, respectively. frequency
represents the sum of the frequencies of the class IDS tested by the
subtree relative to the sum of the frequencies of all class IDSs, and it
weights the expected time overhead of trees vs. arrays. space-speed-
trade-off is used to convert overhead in terms of space into
equivalent units of time overhead, representing the balance between
the importance of saving time vs. the importance of saving space.
In our implementation, space-speed-trade-off is 0.15, indicating that an
incremental increase in space cost of a factor of \(X\) is matched by
an incremental increase in speed of a factor of 0.15X; in our system,
we consider speed benefits to be nearly 7 times more important than
space costs.

The results of this second phase of the tree-construction algorithm are shown in Figure 6, where the largest subtree replaced by a single
array lookup is circled, for \(\text{n.system's assumptions about the}
relative time and space costs of equality and less-than tests versus
array lookups.\)

This bottom-up pass over the binary tree constructed by the
previous phase, selectively replacing subtrees by arrays, has the advantage of always improving the implementation of the dispatch
tree (under assumptions of correctness of the time and space
estimates) and taking time linear in the size of the original binary
tree. However, it only considers those few subranges of class IDSs
that correspond to subtrees, and many subranges that could make
better arrays are not considered. Future work includes studying
algorithms that better integrate selection of good array subranges
with decision tree construction.

4.3 Comparison with Previous Work

Many standard strategies for single dispatching were described at
the start of section 4. However, we are not aware of any previous
work that blends any combination of linear searching, binary
searching, and array lookups in a single dispatching strategy. All
previous work studied a single strategy, applied universally across
dispatch sites.

Some previous work has used subclass tests in dispatching [Chen et
al. 94]. Our algorithm does not use subclass tests, but instead uses
combinations of less-than tests on class IDSs to approximate
combinations of subclass tests. Subclass tests require only
from the constructed lookup DAGs directly into C or
translations from the constructed lookup DAGs directly into C or
SPARC assembly code and indirectly through Vortex intermediate
code. This last option supports applying Vortex’s optimizations to
the dispatch functions themselves, enabling, for example, inlining
of short callee methods into the dispatching function itself [Rose
88, Zendra et al. 97]. In the experimental results that follow, we
focus our study on the approach of direct translation to SPARC
assembly code.

In assessing the effectiveness of our algorithm, we are primarily
concerned with how well our algorithm performs in practice on
large, real programs. We applied our algorithm to the collection of
benchmark programs described in Table 1. These programs make
heavy use of multiple dispatching and light use of predicate classes;
Table 1: Benchmarks

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Size (app. lines + library lines)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>instr-sched</td>
<td>2,400 + 10,700</td>
<td>global instruction scheduler</td>
</tr>
<tr>
<td>typechecker</td>
<td>20,000 + 10,700</td>
<td>typechecker for old Cecil type system¹</td>
</tr>
<tr>
<td>tc2</td>
<td>23,500 + 10,700</td>
<td>typechecker for new Cecil type system¹</td>
</tr>
<tr>
<td>compiler</td>
<td>50,000 + 10,700</td>
<td>old version of Vortex optimizing compiler</td>
</tr>
</tbody>
</table>

¹. The two typecheckers are separate pieces of code, using different data structures and algorithms, and were written by different people.

Table 2: Lookup DAG Statistics

<table>
<thead>
<tr>
<th>compiler</th>
<th>minimum</th>
<th>median</th>
<th>average</th>
<th>maximum</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td># of methods per generic function</td>
<td>0</td>
<td>1</td>
<td>1.8</td>
<td>542</td>
<td>12,412</td>
</tr>
<tr>
<td># of expressions per generic function</td>
<td>0</td>
<td>1</td>
<td>0.96</td>
<td>5</td>
<td>6,556</td>
</tr>
<tr>
<td>dynamically typed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of interior nodes</td>
<td>0</td>
<td>1</td>
<td>1.1</td>
<td>172</td>
<td>7,299</td>
</tr>
<tr>
<td># of edges</td>
<td>0</td>
<td>2</td>
<td>3.0</td>
<td>793</td>
<td>20,555</td>
</tr>
<tr>
<td>avg. path length</td>
<td>0</td>
<td>1</td>
<td>0.95</td>
<td>4.0</td>
<td>20,555</td>
</tr>
<tr>
<td>max. path length</td>
<td>0</td>
<td>1</td>
<td>0.96</td>
<td>5</td>
<td>6,556</td>
</tr>
<tr>
<td>statically typed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of interior nodes</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
<td>167</td>
<td>2,248</td>
</tr>
<tr>
<td># of edges</td>
<td>0</td>
<td>0</td>
<td>1.3</td>
<td>616</td>
<td>8,713</td>
</tr>
<tr>
<td>avg. path length</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>3.0</td>
<td>8,713</td>
</tr>
<tr>
<td>max. path length</td>
<td>0</td>
<td>0</td>
<td>0.23</td>
<td>5</td>
<td>2,248</td>
</tr>
</tbody>
</table>

Cecil is the language supporting the largest subset of the predicate dispatching model in which large benchmark programs are available. Several previous studies on multimethod dispatching have used data drawn from some of these benchmarks.

The next three subsections study different aspects of our algorithm: subsections 5.1 and 5.2 examine the structure and size of the lookup DAGs and dispatch trees, respectively, produced by our algorithm, while subsection 5.3 presents data on the bottom-line speed of the dispatch functions produced by our algorithm. We do not study dispatch function construction time in detail, but constructing dispatchers is fast in practice and a negligible fraction of the overall time to compile a program.

5.1 Lookup DAG Results

Our first experiment studies the structure of lookup DAGs produced for the generic functions in the benchmark programs. We report the results only for the largest program, compiler; the smaller programs produced simpler lookup DAGs. For each generic function, we computed the number of methods in the generic function, the number of tested expressions (dispatched formals and predicate class test expressions) in the generic function, the number of interior nodes, the number of edges (merging edges with the same source and target node), and the average (not weighted by frequency) and maximum path length through the lookup DAG to a non-error leaf node, where sensible, we also computed the total of these values across all generic functions. We simulated two versions of each program: one where no static class information was available (all type declarations are ignored, and all classes are assumed possible at all tested expressions, except for predicate class test expressions which are known to return a boolean) and one where static class information is assumed to be sufficient to ensure that no message not-understood errors can occur.

The results for the compiler benchmark are shown in Table 2. This program has 861 concrete classes (57% of which had multiple inheritance somewhere in their ancestry) and 6,860 generic functions. Most generic functions are small, with a couple of methods and typically a single dispatched expression. But some generic functions are quite large (5 generic functions have more than 100 methods in them) and some generic functions have several dispatched expressions (312 generic functions have 2 tested expressions, 63 have 3, 11 have 4, and 3 have 5 tested expressions). The number of interior states and edges in the constructed DAGs correlates with the structure of the generic functions; the statically typed versions for small generic functions are simplified by ruling out the message-not-understood error case. For the more complex generic functions, the average path length is below the maximum path length, showing benefits for not dispatching on all tested expressions.

5.2 Dispatch Tree Results

Our second experiment studies the structure of the dispatch trees produced for the interior nodes in the lookup DAGs for the benchmark programs. Again, we report results only for the largest program, compiler. To compare the effectiveness of our algorithm mixing array lookups, equal tests, and less-than tests (named =, <, [] in tables) against simpler, less flexible algorithms,
we also implemented four restricted versions of our algorithm, using only array lookups (named 1), only equal tests (implementing linear search as in polymorphic inline caches [Hölze et al. 91]) (named =), only less-than tests (implementing binary search as in TWIST [Nakamura et al. 96] and Smalltalk [Zendra et al. 97]) (named <), and only equal and less-than tests (implementing the binary tree subtree of our algorithm from section 4.2.1 without the array replacement subtree from section 4.2.2) (named =, <). We studied both dynamically and statically typed versions of the benchmark programs. We also studied deriving class frequency distributions from both dynamic profile data and static estimates.

For each dispatch tree to be constructed, we compute the range of class IDs, the number of mapped class IDs in this range (ignoring impossible class IDs), the number of intervals, and ratio of the number of intervals to the number of target nodes mapped by these class IDs. For each of the five construction algorithms (our full algorithm and the four restrictions), we compute the number of test nodes in the constructed dispatch tree (broken down into the number of equal, less-than, and array-lookup test nodes in the tree), the size in bytes of the machine code and data to represent the dispatch tree, and the expected average time cost in cycles for dispatching. We also compute the total number of the different kinds of tests and the total space cost over all dispatch trees in the program.

The results for the compiler benchmark are shown in Table 3. (We show only dynamic-profile-guided results; results guided by static estimates were nearly the same in terms of these space-oriented statistics.) In the dynamically typed versions, all expressions corresponding to predicate class boolean tests have only 2 possible classes, while all formal expressions have all 861 possible classes. The total space cost for the 7299 decision trees (the vast majority of the overall space cost of the dispatch functions) in the dynamically typed versions is usually near 200KB, but pure linear search consumes over 1MB of space and pure arrays consume over 6MB of space. The total space cost for the 2248 decision trees in the statically typed versions is usually under 50KB, but again pure linear search and pure arrays perform relatively poorly.

5.3 Performance Results

Our final experiment assesses the bottom-line impact of our dispatching algorithm on the execution speed of all of our benchmark programs. To gain a realistic estimate of the impact of our optimizations, we first apply all of Vortex's aggressive static and profile-guided optimizations to try to optimize dynamic dispatches without recourse to the run-time dispatching mechanism; only those dispatches that could not otherwise be optimized invoke our generated dispatch functions. We measure the performance of each of our benchmarks under each of our five construction algorithms, with static and dynamic typing, and with profile-driven and estimated frequency distributions. As a point of rough comparison, we also report the performance of Vortex's existing polymorphic inline cache-based dispatchers (PICs). Vortex's PICs have an advantage over our new generated dispatchers by being constructed for each call site separately, specialized to the classes occurring at run-time at each call site, but they are worse at handling multiple dispatching and predicate classes than our new generated dispatchers.

The results for the benchmarks are presented in Table 4, as whole-program speed-ups relative to the PIC-based implementation. Our dispatching functions yield speed improvements over the previous PIC strategy of up to 30%, with bigger speed-ups accruing to bigger programs. Our flexible dispatching strategy mixing equal, less-than, and array-based lookups generally produces the best speed-ups, although purely binary search using less-than tests works nearly as well. Linear search using equal tests performs poorly without dynamic profile information, as expected, but the other techniques are less dependent on dynamic profile guidance. Purely array-based lookups were worse than the other techniques. Assuming static type checking had some benefit, particularly in the tc2 benchmark.

6 Conclusions and Future Work

We have developed a new algorithm for building message dispatch functions, supporting the general model of predicate dispatching. Our algorithm includes novel approaches to the implementation of multiple and single dispatching. Our algorithm is generally more flexible than previous algorithms for multiple dispatching, allowing, for instance, customizing the order of checking formal parameters and specializing the set of formals needing checks along different paths in the lookup DAG. It also is more flexible than previous algorithms for single dispatching, allowing mixing different kinds of basic tests into a combined dispatch tree, separately for each single dispatch. Our algorithm is unusual in taking into account both static class information and dynamic profile information (when available) to help construct faster, smaller dispatchers. We have assessed our algorithm on a collection of large Cecil programs, using a compiler incorporating a suite of static and profile-guided message optimizations, demonstrating the bottom-line effectiveness of our new algorithms.

Our current experiments have studied the problem of constructing a dispatch function given complete knowledge of the methods in each generic function and the program's whole class hierarchy, as would occur in a whole-program optimizer or dynamic compiler. In the future we plan to study constructing dispatchers by quickly splicing together fragments computed from partial views of the program (such as could be done in a separate-compilation-based environment), to study constructing and adapting dispatching functions on-line as the program runs (exploiting on-line rather than off-line knowledge of the program's execution profile), and to study selective specialization of dispatch functions for the more specific characteristics found at particular groups of similar call sites. We also wish to change the Vortex compiler's message send optimizations to be guided more systematically and generally by lookup DAGs constructed by our algorithm.

Acknowledgments

Michael Ernst in particular but also Craig Kaplan, David Grove, Todd Millstein, Jonathan Nowitz, and Jonathan Aldrich contributed to this work in early discussions about more general and flexible dispatching algorithms. Vassily Litvinov and Todd Millstein provided useful comments on earlier drafts of this paper. Richard Ladner developed several optimal polynomial time algorithms for restricted cases of the dispatch tree construction problem. Jonathan Bachrach and Wade Holst discussed with us general issues and techniques for multimethod dispatching.

This research is supported in part by an NSF grant (number CCR-9503741), an NSF Young Investigator Award (number CCR-9457767), and gifts from Sun Microsystems, IBM, Xerox PARC, Object Technology International, Edison Design Group, and Pure Software.

1 Execution times were measured on UltraSPARC-1/170 workstations with ~200MB RAM, taking the median time of 11 runs of the benchmark. On this hardware, median times still vary by a few percent, so small differences in computed speed-up should not be considered significant.
Table 3: Dispatch Tree Statistics

<table>
<thead>
<tr>
<th>compiler</th>
<th>minimum</th>
<th>median</th>
<th>average</th>
<th>maximum</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of class IDs</td>
<td>2</td>
<td>861</td>
<td>861</td>
<td>861</td>
<td></td>
</tr>
<tr>
<td>count of class IDs</td>
<td>2</td>
<td>861</td>
<td>861</td>
<td>861</td>
<td></td>
</tr>
<tr>
<td># of intervals</td>
<td>2</td>
<td>3</td>
<td>4.8</td>
<td>585</td>
<td></td>
</tr>
<tr>
<td>intervals per target</td>
<td>1</td>
<td>1.5</td>
<td>1.8</td>
<td>9.5</td>
<td></td>
</tr>
</tbody>
</table>

### Dynamically Typed

<table>
<thead>
<tr>
<th>=, &lt;, []</th>
<th># of = tests</th>
<th>0</th>
<th>2</th>
<th>2.5</th>
<th>24</th>
<th>18,718</th>
</tr>
</thead>
<tbody>
<tr>
<td># of &lt; tests</td>
<td>0</td>
<td>1</td>
<td>1.4</td>
<td>22</td>
<td>10,151</td>
<td></td>
</tr>
<tr>
<td># of [] tests</td>
<td>0</td>
<td>0</td>
<td>0.42</td>
<td>4</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td>size</td>
<td>2</td>
<td>18</td>
<td>26.</td>
<td>867</td>
<td>190,054</td>
<td></td>
</tr>
<tr>
<td>avg dispatch time</td>
<td>7.5</td>
<td>8.5</td>
<td>8.9</td>
<td>17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| =, < | # of = tests | 1 | 2 | 2.8 | 269 | 20,364 |
| # of < tests | 0 | 1 | 1.6 | 325 | 11,680 |
| size (bytes) | 2 | 18 | 26. | 867 | 190,054 |
| avg dispatch time | 2.5 | 8.5 | 8.9 | 17 | |

| = | # of = tests | 1 | 2 | 29. | 859 | 213,553 |
| size (bytes) | 2 | 18 | 26. | 867 | 1,008,595 |
| avg dispatch time | 2.5 | 8.0 | 8.7 | 25 | |

| < | # of < tests | 0 | 2 | 3.8 | 583 | 27,837 |
| size (bytes) | 2 | 18 | 28. | 867 | 205,610 |
| avg dispatch time | 2.5 | 9.8 | 10. | 17 | |

| [] | # of [] tests | 1 | 8 | 867 | 6,136,676 |
| size (bytes) | 8.0 | |
| avg dispatch time | 8.0 | |

### Statically Typed

| range of class IDs | 2 | 64 | 254. | 861 | |
| count of class IDs | 2 | 8 | 126. | 861 | |
| # of intervals | 2 | 3 | 4.7 | 585 | |
| intervals per target | 1 | 1 | 1.2 | 8.5 | |

| =, <, [] | # of = tests | 0 | 1 | 1.4 | 17 | 3,084 |
| # of < tests | 0 | 1 | 1.2 | 16 | 2,616 |
| # of [] tests | 0 | 0 | 0.15 | 5 | 326 |
| size (bytes) | 2 | 13 | 21. | 867 | 47,152 |
| avg dispatch time | 2.5 | 5.8 | 6.5 | 12 | |

| =, < | # of = tests | 0 | 1 | 2.3 | 269 | 5,094 |
| # of < tests | 0 | 1 | 2.0 | 325 | 4,429 |
| size (bytes) | 2 | 13 | 21. | 867 | 47,152 |
| avg dispatch time | 2.5 | 5.8 | 6.5 | 12 | |

| = | # of = tests | 0 | 3 | 76. | 859 | 171,456 |
| size (bytes) | 2 | 16 | 122. | 883 | 274,916 |
| avg dispatch time | 2.5 | 7.2 | 6.9 | 22 | |

| < | # of < tests | 0 | 2 | 3.6 | 583 | 8,179 |
| size (bytes) | 2 | 13 | 19. | 867 | 43,322 |
| avg dispatch time | 2.5 | 6.4 | 6.7 | 13 | |

| [] | # of [] tests | 1 | 8 | 867 | 584,305 |
| size (bytes) | 70 | 260. | 867 | 584,305 |
| avg dispatch time | 8.0 | |
### Table 4: Benchmark Performance Measurements

<table>
<thead>
<tr>
<th>benchmark</th>
<th>version</th>
<th>dynamically typed</th>
<th>statically typed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>estimated</td>
<td>dynamic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>profiles</td>
<td>profiles</td>
</tr>
<tr>
<td>instr-sched</td>
<td>=, &lt;, []</td>
<td>1.08</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>=, &lt;</td>
<td>1.09</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>=</td>
<td>0.86</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>&lt;</td>
<td>1.12</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>[]</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>typechecker</td>
<td>=, &lt;, []</td>
<td>1.12</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>=, &lt;</td>
<td>1.11</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>=</td>
<td>0.81</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>&lt;</td>
<td>1.12</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>[]</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>tc2</td>
<td>=, &lt;, []</td>
<td>1.13</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>=, &lt;</td>
<td>1.14</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>=</td>
<td>0.85</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>&lt;</td>
<td>1.16</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>[]</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td>compiler</td>
<td>=, &lt;, []</td>
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<td>1.26</td>
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1. Profile information has no effect on purely array-based lookups.

### References


